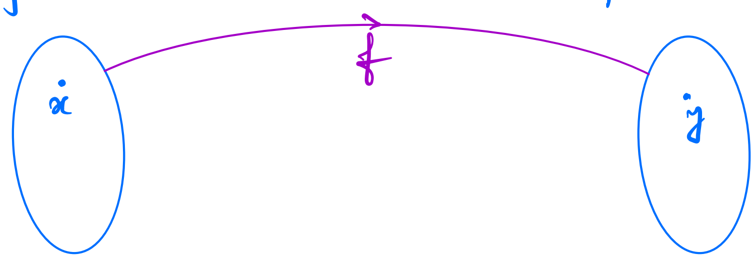
⊛ A **function** is a relation between a set of inputs & a set of **permissible** outputs.

meaning every element in the input set has one & only one image in the output set

Note: We call functions as mappings too, as it maps one element of Input set to one-unique element in the Output set.

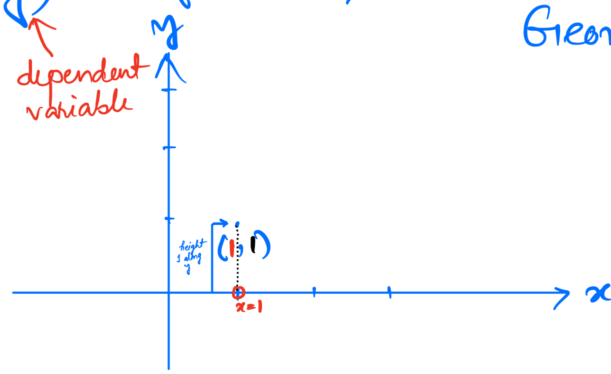
Notation :- ① Input Set   Output Set



$y = f(x)$ ⇝ 'y' is equal to 'f' of 'x'.

② (x, y) or $(x, f(x))$ ⇝ More used for Geometry.

independent variable   dependent variable



⊛ The set of inputs is called the **Domain** & the set of outputs is called the **Range** of the function.

Note: The is something called Codomain, it contains the Range of the function as a set.

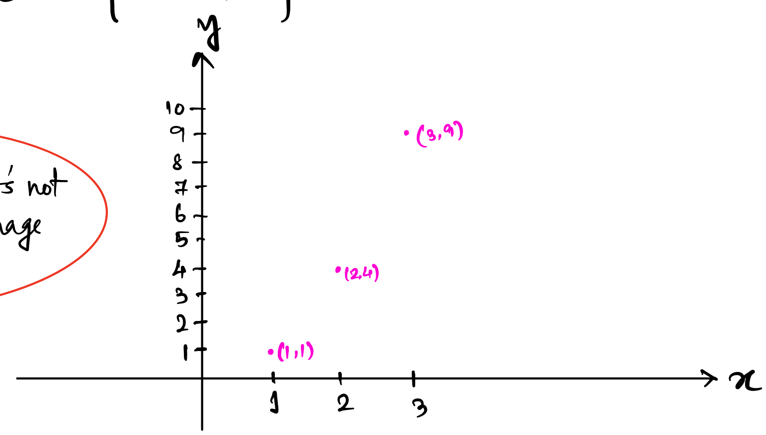
⊛ **Graph of a Function:** It's the geometry associated to the function.

Say, Domain $= \{1, 2, 3\}$

Codomain $= \mathbb{R}$, the set of real numbers.

Range $= \{1, 4, 9\}$

In this notation it's not clear what's the image of 1 or 2 or 3



If we write $(1,1)$, $(2,4)$, $(3,9)$, then it makes complete sense to draw, as we don't know the exact function.

⊛ **Set Builder Notation:-** $\{x \mid x \text{ has some property}\}$

'x' such that 'x has some prop.'

Example: ① $\{x \mid 1 < x < 5\}$

② $\{x \mid 1 \leq x \leq 5\}$

③ $\{x \mid x \geq 0\}$

④ $\{x \mid x \text{ is any real number}\}$

$\{x \mid x \in \mathbb{R}\}$

$\in$ ↳ Belongs to

⊛ **Interval notation:** $(a, b) = \{x \mid a < x < b\}$ ⟶ Open Interval

$(a, b] = \{x \mid a < x \leq b\}$

$[a, b) = \{x \mid a \leq x < b\}$

$[a, b] = \{x \mid a \leq x \leq b\}$ → Closed Interval.

⊛ **Evaluating Functions:** If $f(x) =$ some formula of x, then to evaluate, replace x by the given point where you are asked to evaluate.

Eg: $f(x) = x^2 + 4x + 3$, evaluate at $x = 1$, a, $b+1$

$f(1) = 1^2 + 4 \cdot 1 + 3 = 1 + 4 + 3 = 8$

$f(a) = a^2 + 4a + 3$

$$f(b+1) = (b+1)^2 + 4(b+1) + 3$$
$$= (b^2 + 2b + 1) + (4b + 4) + 3$$
$$= b^2 + 6b + 8$$

⊛ Finding Domain.

① $f(x) = \dfrac{3}{x^2 - 1}$

We cannot divide by 0

So, Domain$(f) = \{x \mid x^2 - 1 \neq 0\} = \{x \mid x \neq \pm 1\}$

② $f(x) = \dfrac{2x+5}{3x^2+4}$

Since $3x^2 + 4 \geq 4$, for all real values of x.

So, Domain$(f) = (-\infty, \infty) = \mathbb{R}$

③ $f(x) = \sqrt{4 - 3x}$.

↑

Part under the square root symbol must be non-negative.

So, Domain$(f) = \{x \mid 4 - 3x \geq 0\}$
$$= \{x \mid 4 \geq 3x\}$$
$$= \{x \mid \tfrac{4}{3} \geq x\}$$
or
$$\{x \mid x \leq \tfrac{4}{3}\}$$

⊛ Function Notation:

– by table

Hours after Midnight	Temperature (°F)	Hours after Midnight	Temperature (°F)
0	58	12	84
1	54	13	85
2	53	14	85
3	52	15	83
4	52	16	82
5	55	17	80
6	60	18	77
7	64	19	74
8	72	20	69
9	75	21	65
10	78	22	60
11	80	23	58

Table 1.1 Temperature as a Function of Time of Day

– by graph

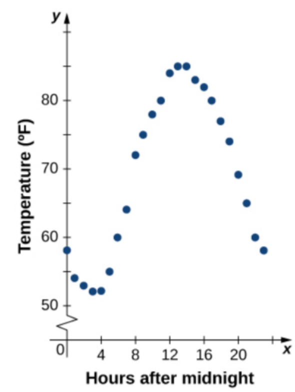


Figure 1.6 The graph of the data from **Table 1.1** shows temperature as a function of time.

– by formula.

$$f(x) = \sqrt{x+3} + 1$$

Practice : Find the $\begin{cases} \text{domain of } f. \\ \text{zeros of } f \text{ (if any).} \end{cases}$

& try sketching it.

⊛ Piecewise-defined function.

$$f(x) = \begin{cases} \text{Rule 1 for } x, & \text{when } x \text{ is in some set } A_1 \\ \text{Rule 2 for } x, & \text{when } x \text{ is in some other set } A_2 \\ \vdots & \end{cases}$$

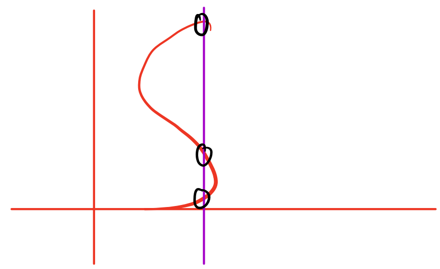
& so on.

Eg: $f(x) = \begin{cases} -x, & \text{when } x < 4 \\ 0, & \text{when } x = 4 \\ x^2, & \text{when } x > 4 \end{cases}$

Try to sketch.

⊛ Vertical Line Test: Given a function f, every vertical line that may be drawn intersects the graph of f, no more than once.

If it intersects more than once, then the set of points doesn't represent a function.



⊛ Zeros or x-intercepts & y-intercepts of a function.

$f(x) = -x+2$ ⇝ only one zero at $x=2$.
⇝ y-intercept is given by $(0, f(0))$
ie, $(0, 2)$

⊛ Increasing & Decreasing function over an interval.

- Increasing $x_1, x_2 \in I$ s.t $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$
- Strictly increasing $x_1, x_2 \in I$ s.t $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

⊛ Composition & Combining functions.

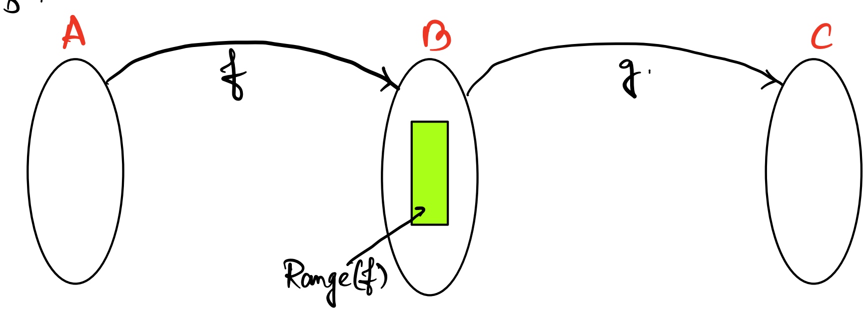
Composition: If we have such situation, we have

$f: A \longrightarrow B$ is a function & $g: B \longrightarrow C$ is a function

such that range $(f) \subseteq$ domain (g).
subset

Then can we define a single function between A & C?

Yes, we can. It's called the composition of two functions.



Written by $\boxed{g \circ f}$ (said, g compose f)

Note: f gets the values first from x
g gets the values then from $f(x)$.

So, $(g \circ f)(x) = g(f(x))$

Similarly, $(f \circ g)(x) = f(g(x))$.

<u>Note</u>: $(g \circ f)(x) \neq (f \circ g)(x)$ in general.

<u>Eg</u>. $f(x) = x^2$, $g(x) = 3x+1$

$$(f \circ g)(x) = f(g(x)) = f(3x+1)$$
$$= (3x+1)^2$$

& $$(g \circ f)(x) = g(f(x)) = g(x^2)$$
$$= 3(x^2) + 1.$$

⊛ Algebra of Functions:

$$(f+g)(x) = f(x) + g(x) \longrightarrow \text{Sum}$$

$$(f-g)(x) = f(x) - g(x) \longrightarrow \text{Difference}$$

$$(f \cdot g)(x) = f(x) \cdot g(x) \longrightarrow \text{Product}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ for } g(x) \neq 0 \longrightarrow \text{Quotient}$$

⊛ Even Functions & Odd Functions:

Even: $f(x) = f(-x)$, for all x in the domain.

Odd: $f(-x) = -f(x)$, for all x in the domain.

⊛ Absolute Value Functions:

$$f(x) = |x| \text{ or } \operatorname{abs}(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$